Bursts and avalanches – the dynamics of polycentric urban evolution

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Abstract

Urban construction activities are subject to periods of fast expansion followed by periods of slow growth. Some of these expansions are limited is size, while other are huge. Therefore, it is not surprising that equilibrium oriented classical models of urban spatial structure are hard pressed to explain the formation of modern cities with polycentric structure and births of sub-centers in particular.

To understand the development of cities' spatial pattern we develop a model of urban spatial dynamics that is driven by real estate entrepreneurs of two types that differ in the degree of risk aversion. The developers act in light of city planning committee that formulates urban development policy. Its salient feature is the time lag between the moment of purchase of property rights by land developers until the realizations of revenues. We assume that this lag varies in space and can be reduced in case of high demand for dwellings.

With the model we demonstrate how the interaction between demand for dwellings, developers' choices and planning policies lead to the creation of new urban sub-centers. Model dynamics is characterized by long out of equilibrium periods followed by sudden bursts of construction activity that resembles self-organized criticality (SOC).

1. Introduction

At the backdrop of the spatial dynamics of cities there is an adverserial interaction of land developers and planning authorities. Urban planners seek to advance the public's interest by efforts to prevent sprawl and to ensure accessibility to open spaces. This they do by promoting dense settlements by restricting the locational and other decisions of developers. Among other things, planners determine the location and intensity of land-uses by specifying types of building that can be constructed within the municipal territory. In some places building activity is not allowed. At some locations restricted construction activity is permitted. The relaxation of building restrictions involves concentrated effort by developers and prolonged procedures. The result is the issuance of building permits and consequent varying number of building starts (EUROSTAT, 2010, US Census Bureau, http://www.census.gov/const/www/newresconstindex.html).

The adversity between planning authorities and developers is not uniform in space and time. In cities in which population growth is stagnat and during periods of macroeconomic downturns low demand leads to lethargic behavior of developers. Some developers utilize such periods to purchase land parcels for future development, including land that is not zoned for development. Periods of rapid population growth and economic expansions create high demand and consequent housing price bubbles (Glaeser et al, 2008), fervent search by developers for all available, developable land parcels and eventual construction booms. Thus for example, the immigration to urban centers in USA starting in the mid-1960s and vast immigration wave from the former Soviet Union to Israel in the early 1990's caused scrambling behavior by developers, housing price volatility (Benchetrit et al 2008), construction booms, urban economic restructuring and changes in settlements patterns (Waldinger, 1989; Spelman, 2009; Beltratti et al, 2010).

However, aggregate building starts data tell an incomplete story. Casual examination of figures 1 and 2 below suggests that the combined effects of macro-economic cycles and immigration flows tell a partial story only. Thus, although the effect of immigration from the former Soviet Union on Israeli cities and of the 2007 downturn in the USA is easily recognizable in the charts, there are clear local trends that do not seem to be related to these phenomena. It seems that there are local forces generating additional, more refined, cycles of fast growth followed by periods of slow growth. Furthermore, these building starts data say nothing about the spatial incidence of these trends. There is growing evidence that the speed of spatial expansions in some cities displays irregular patterns (Liu et al, 2011) and that the process occurs at very different rates in different zones of cities (Cao et al, 2011).

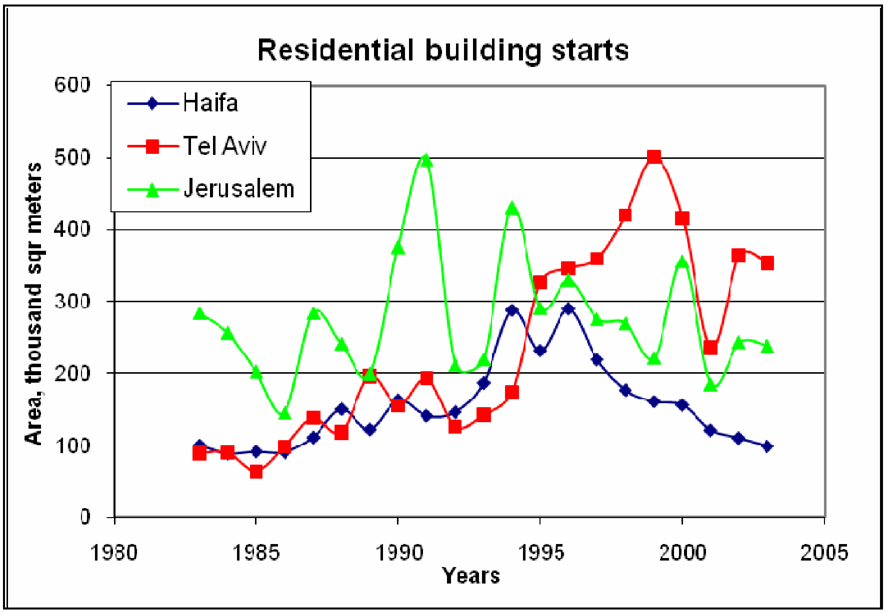
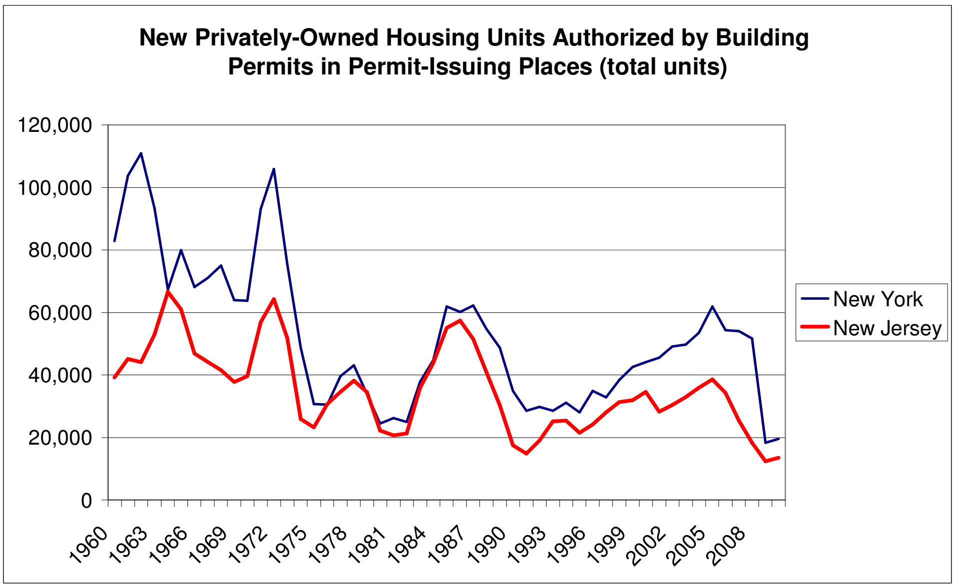


Figure 1: Building starts in major Israeli cities 1980 – 2005
[Source: Israel Central Bureau of Statistics]



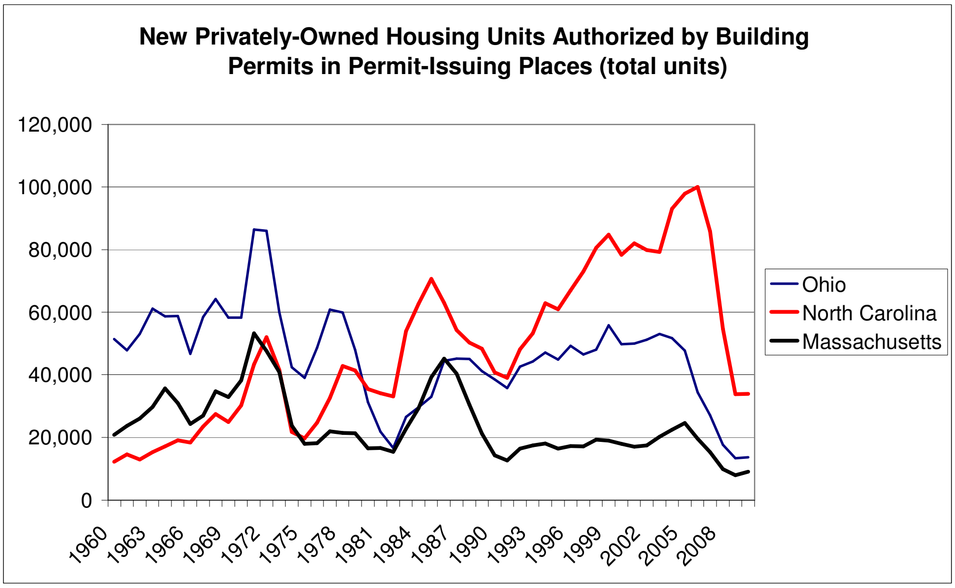


Figure 2: Building starts in selected USA states 1960-2010
[Source: http://www.census.gov/const/www/newresconstindex.html]

At least in part the fact that construction activities are subject to periods of fast expansions followed by periods of slow growth can be explained by variations in demand and willingness to pay that recurrently pushes the city as a whole away from an equilibrium state. Therefore, it is not surprising that equilibrium oriented classical models of urban spatial structure are hard pressed to explain the major emerging feature of the modern cities, namely the births of urban sub-centers and the resulting polycentric urban pattern. In fact, there is a paucity of models that are concerned with both the spatial and temporal evolution of modern, polycentric cities.

In this paper we suggest that beyond the familiar, global economic and demographic trends, urban spatial expansion dynamics are significantly influenced and fashioned by local forces. More specifically, we suggest that the interaction between municipal planning policies and profit-maximizing choices of investors in real estate projects in light of varying demand for dwellings in a city may explain much of the spatial-temporal evolution of polycentric cities.

In our analysis we are inspired by the possibility that the formation of urban spatial pattern is similar to the phenomenon of Self-Organized-Criticality (SOC). SOC was developed in order to describe the dynamics of non-equilibrium systems that respond to external perturbations with events of all sizes at no apparent characteristic scale. According to Per Bak "… systems evolve to the complex critical state without interference from any outside agent …Large catastrophic events occur as a consequence of the same dynamics that produces small ordinary everyday events" (Bak, 1996). The sand-pile metaphor is commonly used to illustrate conceptually SOC ideas (Bak, 1996; Christensen & Moloney, 2005). A wide range of SOC applications were developed in fields as diverse as physics (Christensen & Moloney, 2005, Dickman et al, 2000, Dhar, 2006; Bak et al, 1987, 1988; Bak, 1990, Carlson & Swindle, 1995), time series analysis (Xianzhong & Sheng, 2010) and cellular automata models (Bak et al 1989, Alstrom & Leao, 1994; Sales, 1993).

Power law is a strong sign of SOC. For example, the sizes of the sand-pile avalanches follow this law (Bak, 1996). Some indications of SOC in the case of cities were obtained by means of the power law and of the fractal structure of the urban built pattern (Batty & Xie, 1999; Benguigui et al, 2000, Chen & Zhou, 2008). Our view of the relation between the SOC and urban dynamics is based on the Gabaix model of proportional growth of the cities (Gabaix, 1999, 2008 and Benguigui et al, 2007). The model assumes that, on average, the growth rate of cities is the same and that it varies randomly in time according to a normal distribution. Gabaix (1999) has shown that in such a system, in a long run, rank size distribution of cities follows the power law. We thus question whether there are internal regularities in developers' behavior that can cause the power-like distribution of the waves of urban growth.

The rest of the paper is comprised of three sections. In the next section we present an agent-based model that generates three dimensional, polycentric urban dynamics. In the following section we present the spatial and temporal evolution of cities generated by the model. In particular, we illustrate waves of spatial expansion that resemble bursts typical of the self-organized criticality. We conclude with discussion of our results.

2. Model concept

Our model describes the city growth as an outcome of the interactions between a planning board and land developers. The population of the city increases and urban planning board fashions land-use plans that defines where construction activities are allowed: Clear-cut administrative measures in the form of zoning restrictions define precisely the location and intensity of construction activities.

2.1. Characteristic Approval Time

Beyond direct statutory enforcement, there are more subtle ways in which city planners influence urban development. Since all construction projects must gain planning approval, the time and effort required of real-state entrepreneurs to achieve building permits at a certain location is a significant factor to be considered. Expected long waiting times, often due to the opposition of environmental lobbyists, may discourage potential entrepreneurs from initiating building projects. Construction permits within areas of municipalities that are zoned for buildings are expected to take shorter time to be issued than permits in other locations.

In general, we assume that in a mono-centric city, the time it takes to obtain a construction approval increases with the distance from the CBD. Furthermore, assuming that the environmental impact of physical structures is proportional to their height, we postulate that at a given location, the approval waiting time will be longer for higher buildings. We combine these assumptions assuming that *Characteristic Approval Time,* (CAT), the time elapsed from the purchase of property rights and until the realization of income from the constructed projects is monotonously increasing with the distance from the CBD and height of the project.

The planning policies implemented by planning boards change occasionally in response to changing conditions. In particular, we assume that as population grows and the city experiences excess demand for housing, the pressure by developers to approve construction, contrary to zoning regulations, can be approved. We introduced changes in planning policies as changes in the CAT.

2.2. Behavior of developers

We assume that there are two types of developers:
Small developers: These are small players that abide strictly by the planning regulations. They purchase land for almost immediate development within existing planning restrictions and not for future development after obtaining land-use variances. And, thus, they do not initiate changes in zoning regulations so as to realize a return on the land they already purchased. In the presence of "small developers" only with a preference for immediate returns combined with CAT that grows with distance from the CBD, one should expect the spatial structure of city to resemble an Alonso gradient.

Big developers: Some developers possess large time impatience. They purchase land for future development in the zones that do not allow building and at a low price. These are speculative purchases. The patient "big developers" will hold the speculatively purchased land until building will be possible after obtaining variance or until land-use plans are changed. In cases that many parcels of speculatively bought land are proximately located in space while the land in the core city is becoming scarce and demand for dwelling units is increasing, the planning authorities may change zoning rules and issue construction permits. After big developers start the construction, developers with a preference for immediate returns on investment will be able to move in and start building as well. As a result one can expect a new urban center to be created.

2.3. Planner's decision-making

The planner establishes zoning and Characteristic Approval Time (CAT) that grows with the distance from CBD. Beyond confounding development to the existing CBD, city planners have other objectives as well. The city's population is growing and over time, the authorities are required to keep housing supply growing as well. City planners monitor the number of households searching for dwellings within city boundaries and construction activity, expressed as number of built dwelling units. If population grows faster than dwelling supply, the excess demand the planner changes CAT or approves construction beyond the existing zones.

3. Model formalization

3.1. *Urban space:*

The city is developing on a square grid of cells, each of which represents a single parcel of land. Developers of all types engage in building activities only if there is excess of demand (i. e., there are currently households without dwelling). Initially, a single cell representing the CBD is located in the center of the grid and the cells within the distance R from the CBD center are available for development. We assume that characteristic approval time τ reflects the municipality counter-sprawl policy and τ increases with the distance from the CBD.

Developers' decision to purchase a land unit is based on two economic parameters: the customers' willingness to pay for a dwelling unit and price of the land parcel. Both decay with the distance from the CBD.

3.2. *Developers' decision-making:*

Formally, impatient developer seeks construction projects that can be implemented in a limited time T that is, the projects for which the characteristic approval time τ is T years or less, $\tau < T$. Since the characteristic time is a function of location and height, the impatient developer seeks parcels for which:

$$\tau(x,h) = \tau_L(x) + \tau_H(h) < T$$

where x represents the distance from the CBD and h is building height.

Although $\tau_L(x)$ is defined by the city planners, $\tau_H(h)$ is an endogenous variable representing decisions of developers and developer can, thus, adjust the height of the building in order to satisfy the T-constraint.

The impatient developer's decision algorithm is thus the following:

1- Choose at random, an available site where $\tau_L(x) < T$
2- Only if a site where $\tau_L(x) < T$ is available, calculate the minimal height $h_{\min}$ required in order to make profit based on the willingness to pay and the land cost. The willingness to pay is defined as $\max_{y \neq x}\left(h_y / d_{xy}^V\right)$ where y symbolizes all cells excepting x itself , h_y is the building height at y, d_{xy} is the distance between parcels x and y, and V is the attraction exponent. In a similar manner, the land cost is calculated as $\max_{y \neq x}\left(h_y / d_{xy}^W\right)$. The exponents used are $V = 0.5$ for WTP and $W = 1$ for the land price.
3- The maximal height of a building that an impatient developer will construct must allow him to wait no more than T time. Solving the equation $\tau(x,h) = \tau_L(x) + \tau_H(h) = T$ for h defines the allowed height. If $h \geq h_{\min}$ the impatient

developer purchases the land, waits time T and builds a building of a height h. The construction itself is assumed to be instantaneous.

4- In all other cases (parcels with $\tau_L(x) < T$ or $h < h_{\min}$), the impatient developer will stay inactive until the next time unit.

The impatient developer's strategy is thus risk-free. They are assured that any project will be approved and finalized in less than T years.

The patient developer has two choices. As long as low characteristic time parcels are available in the urban core he behaves like the impatient developer. He differs from the impatient developer in one respect – the choices of building heights are random (not limited by time constraints). In this case the algorithm is the same for both types of developers. In the case that parcels inside the city are not available anymore to the patient developer speculates and buys land for future construction and future returns in the urban periphery. This case represents speculation because it is against the declared city policy. In order to obtain construction approval, the developer gambles for a future change in the municipality's policy. The patient developer also aims at profitability but accepts time related risks:

1- If a site where $\tau < T$ is not available, search for an area outside the urban core. Such sites can be purchased at a very low price, representing agricultural use. They need to be sufficiently close to developed areas so that demand for urban land uses exist and willingness-to-pay is positive.
2- Calculate attractiveness of every available parcel outside the urban core. The attractiveness is defined by two conditions: Profitability (according to the same WTP and land cost calculations explained above) and the vicinity of at least one parcel already purchased by other patient developer. For a given level of profitability, the parcel with the higher number of neighboring purchased parcels is more attractive.
3- Purchase a parcel x with a probability that is proportional to its attractiveness and wait an undetermined time for building permit. If the construction permit is allowed, develop a project of height $h > h_{\min}$.

Patient developers' speculative behavior grows in volume when sites in the periphery receive building approvals. They are attracted by the peripheral sites since they have justified expectations to receive approvals in that zone, and, indeed, if the dwelling demand is high enough they will be able to develop closer sites. However, financial capabilities of the patient developer are limited. There is an upper bound on the quantity of sites that can be held in a waiting status. In our model we set this limit as four undeveloped parcels for each patient developer.

Clearly, the patient developer takes a different approach from the impatient one. In early stages of city development, when the city's core (i.e. sites where $\tau < T$) is still not fully developed, she will choose sites in it, but since she is not constrained by time, the projects will be considerably more intensive (i.e. high-rise buildings) than those of the impatient developers.

3.3. *Planner's decision-making:*

To decide on zoning or CAT, the model planner estimates urbanization pressure *U(x)* and compares it to the threshold value U_{Th} .The threshold represents the number of high rise buildings that can accommodate population that represents a certain percentage of the city size. If population growths at a rate of r percent, then the threshold for the initial center is $r \cdot \tau$

The urbanization pressure U(x) is calculated as

$$U(x) = \alpha \cdot ED + \beta \cdot P(x)$$

The excess demand for housing ED represents pressure exerted by a growing share of the city's population that is unable to find adequate housing and is expressed by the number of housing units calculated as number of buildings. The development pressure at a certain location $P(x)$ is a function of the number of sites purchased by patient developers beyond the planned urban area. We assume that sites close to concentrations of buildings will have a greater chance to be developed than sites located far away. Also, the higher are the building around the purchased site x, the higher is $P(x)$. The value of $P(x)$ is calculated for every undeveloped parcel x in the city as a weighted average of all neighboring building heights divided by their distance:

$$P(x) = \sum_{i \in N} h_i / d_{ix}$$

Where N is the set of cells at a distance of 6 units, h_i is their height and d_{ix} their distance from x.

α and β are parameters that are intuitively appealing. α is interpreted as a measure of the sensitivity of the planners to the pressure exerted by the growing population and β as sensitivity of the city planner to big developers' pressure. In our model we perform sensitivity analyses on both α and β

The planner thus acts according to the following algorithm concerning parcels at the city's fringe:

1- If $ED \leq 0$ (the dwelling supply meets or exceeds the existing demand), the planner issues approvals when the waiting time expires only in parcels where $\tau < T$
2- If $0 < ED$ but $\alpha \cdot ED + \beta \cdot P(x) < U_{Th}$ for each x, the planner issues approvals after waiting time expiration everywhere. That includes parcels where $\tau < T$ (urban areas) and parcels purchased by "big" developers.
3- If $U_{Th} < \alpha \cdot ED + \beta \cdot P(x)$, search for the parcel x that holds the highest potential, change zoning there and define characteristic approval time around it irrespective of whether a building exists in its vicinity. At this point the planner establishes a new zoning regulation. The new approved for development zone is circular, its area is proportional to $\alpha \cdot ED + \beta \cdot P(x)$, the characteristic time is lowest in the parcel x and T in the circle perimeter.
4- Once construction by big developer starts, characteristic time is reduced and developers with preference for immediate returns will return to build, creating a new Alonso-type sub-center.

Model results

In this section, we present results of model simulations and highlight particular features by isolating and exaggerating them. Among other things we investigated exponential growth of population as a rate within the interval of 0.01 – 0.02 per iteration. Another parameter that was investigated is the radius of the area around new urbanization seeds where the construction approval is obtained. In what follows we assume that there are many impatient developers with a preference for short-term return on investment and few patient developers who are ready to wait longer periods of time.

The city comprised of impatient developers is the simplest case. Under various sets of the relevant parameters the simulations always result in emergence of a mono-centric city with declining building heights as distance from the CBD increases. The model reaches quickly a stable equilibrium. The city structure is stable and there are no endogenous forces capable of pushing it away in any direction. The planner has no means to accommodate additional population that just increases.

The situation changes once a single patient developer is allowed to play. At the initial stage a mono-centric Alonso-type city is developed around the initial central cell, mostly by the impatient developers. The big developer participates in this process, but in parallel purchases land parcels beyond the zone permitted for construction (a maximal number of four undeveloped parcels at a given time). According to their behavior rules, the probability that the "big" developer will purchase next site close to the previous purchased sites grows with the increase of the number of the purchased sites around, and, thus, the purchases of the big developer are concentrated more than it could be for the purely random purchasing process.

When the existing city area is exhausted the planner is faced with a growing dwelling demand on the one hand and increasing pressure of the big developer for construction approvals on the other. The values of U(x) thus grows for every parcel x and, eventually, exceeds at some location the threshold U_{Th}. The planner issues construction permission at this x, which is usually one of the largest clusters of sites purchased by the big developers. The model is thus periodically driven out of equilibrium - periods of graduate development are interrupted by the bursts of activity of several sizes that occur at unexpected times. These large scale events result in emergence of the new urban sub-centers.

The model dynamics thus resembles the sand-pile SOC metaphor, at least in the sense defined in Frigg (2003). Purchased units are grains of sand and the city is a sand-pile. The patient developers' choices act as the sand grains falling according to self-attracting purchase process. Planner's permission in respect to the growing demand and developers' pressure releases an avalanche.

Let us characterize model dynamics by the number of dwelling units constructed at each time step. Figure 4 present this dynamics in the case of 10 "impatient" and a single "patient" developer for the case of $\alpha = 1/200$ and $\beta = 3/4$. The model dynamics is characterized by periodic raise of demand for housing ED (Figure 5), accumulation of sites purchased by big developer beyond the city boundaries (Figure 6) and urbanization pressure U. Eventually, the urbanization pressure reaches the threshold and the planner approves construction beyond the existing zoning. The approval results in a construction burst that gives rise to a new sub-center (Figure 7, 8).

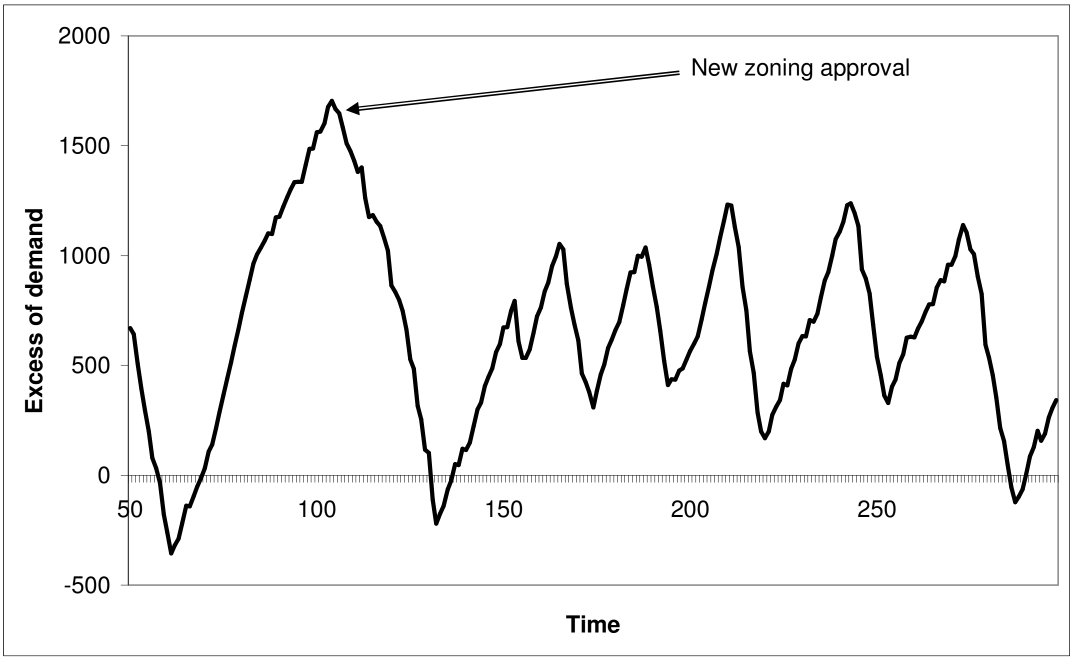


Figure 5: Excess of demand over time

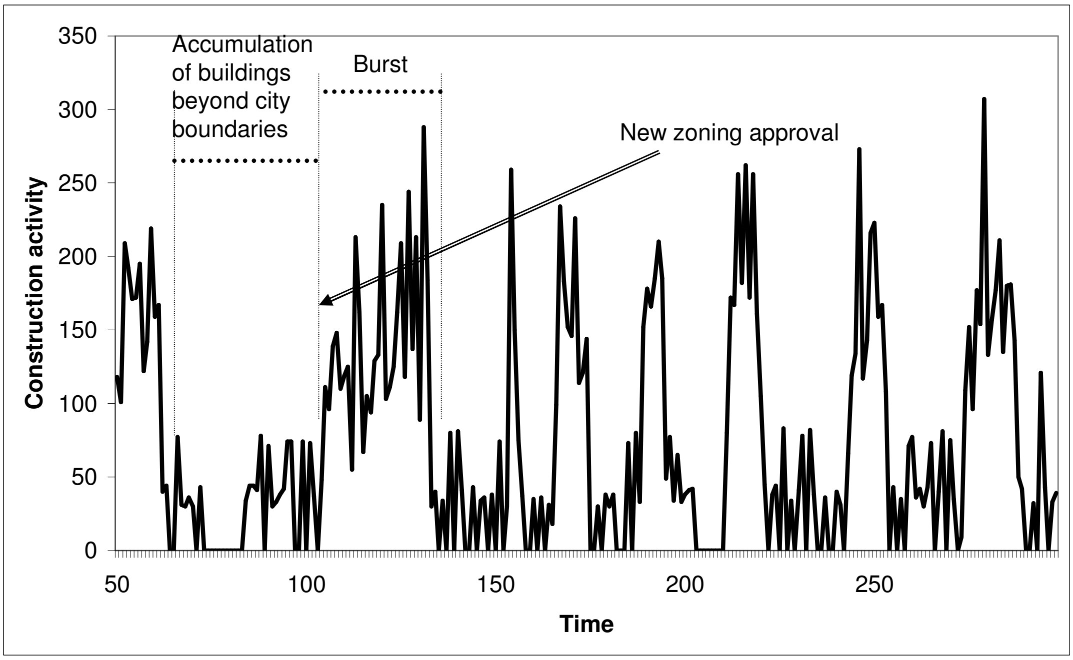


Figure 6: Construction activity as a function of time

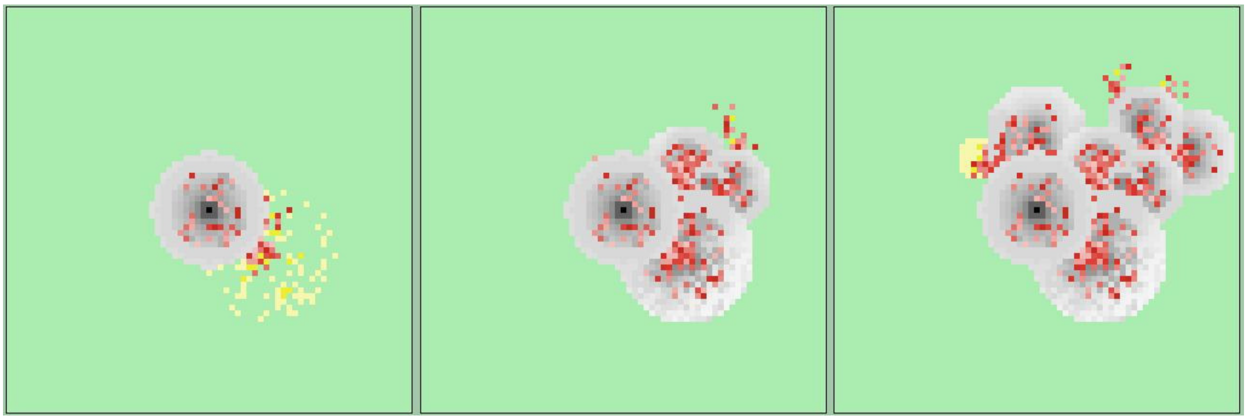


Figure 7: The formation of polycentric urban structure in a model with a 10 impatient and 1 patient developers in a 70 x 70 grid. From left to right, at time 100, 200 and 300

It is noteworthy that the size of the bursts and time intervals between them vary. Some bursts and intervals are relatively large and others are rather small.

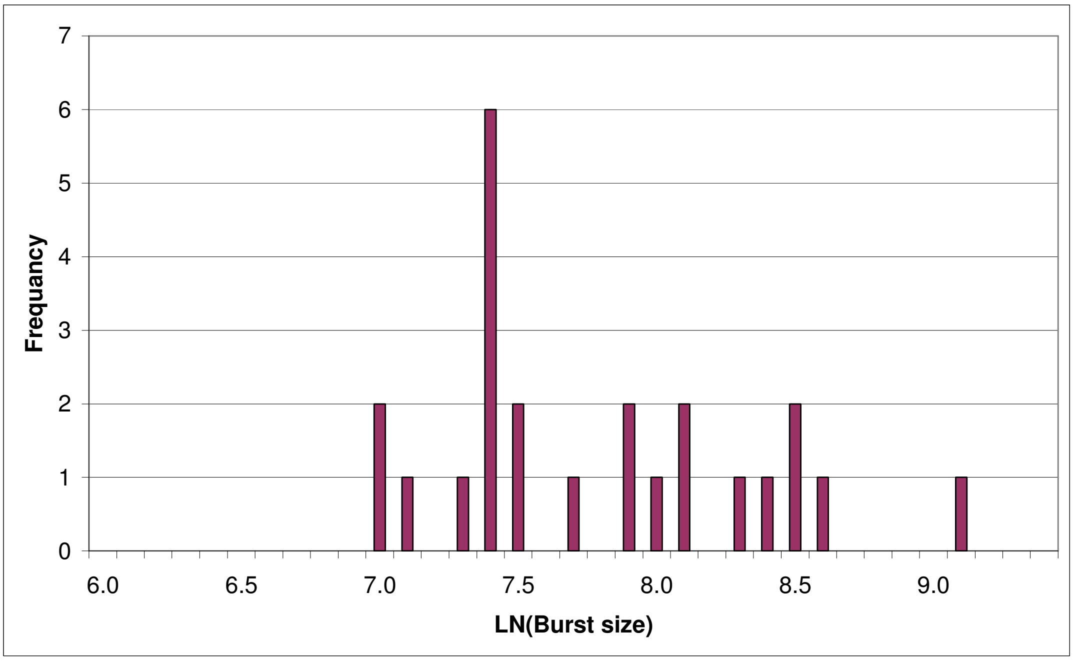


Figure 8: Frequency distribution of bursts

The polycentric structure described above is associated with an unbalanced number of patient and impatient developers (many more impatient developers than patient ones). It is noteworthy that as the share of patient developers increases, the resulting urban pattern is quite different.

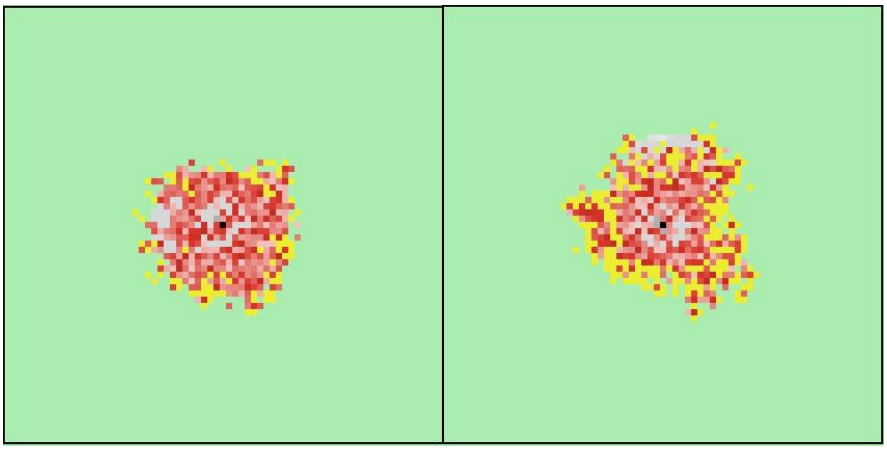


Figure 9: City Spatial structure with a large majority of patient developers

The left side of figure 9 depicts the city structure when 10 impatient and 20 patient developers are present, at time 300. The right side is the resultant structure when 50 patient developers are active with an equal number of impatient developers.

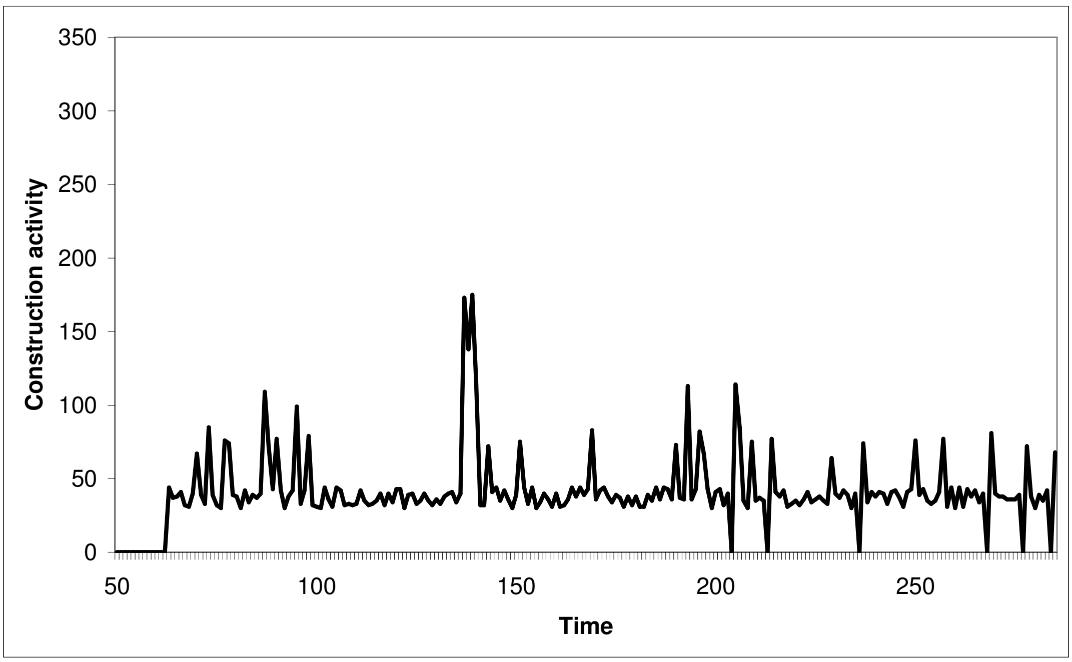


Figure 10: Construction activity when 20 patient and 10 impatient developers are present

The above illustrations are suggestive. They indicate that polycentric city is a continuous evolving structure between two extremes. An ordered world of impatient developers leads to the classical Alonso type city. A completely unordered, or entropic city, is the result of building activity of patient developers only. The polycentric structure that is so familiar to us is the result of a particular mix of the two types of developers. Their numbers suggest that only some pressure exerted by excess demand for housing and by developers seeking economic rents on agricultural lands results in construction activity and the formation of new sub-centers.

Finally, figure 11 presents the rank-size distribution of big bursts in the case of 10 impatient developers and 1 patient one. This distribution follows power law and can thus serve as an evidence of the SOC. While the fit is not perfect it is certainly suggestive. These results suggest that SOC is a valid framework for the analysis of our urban model.

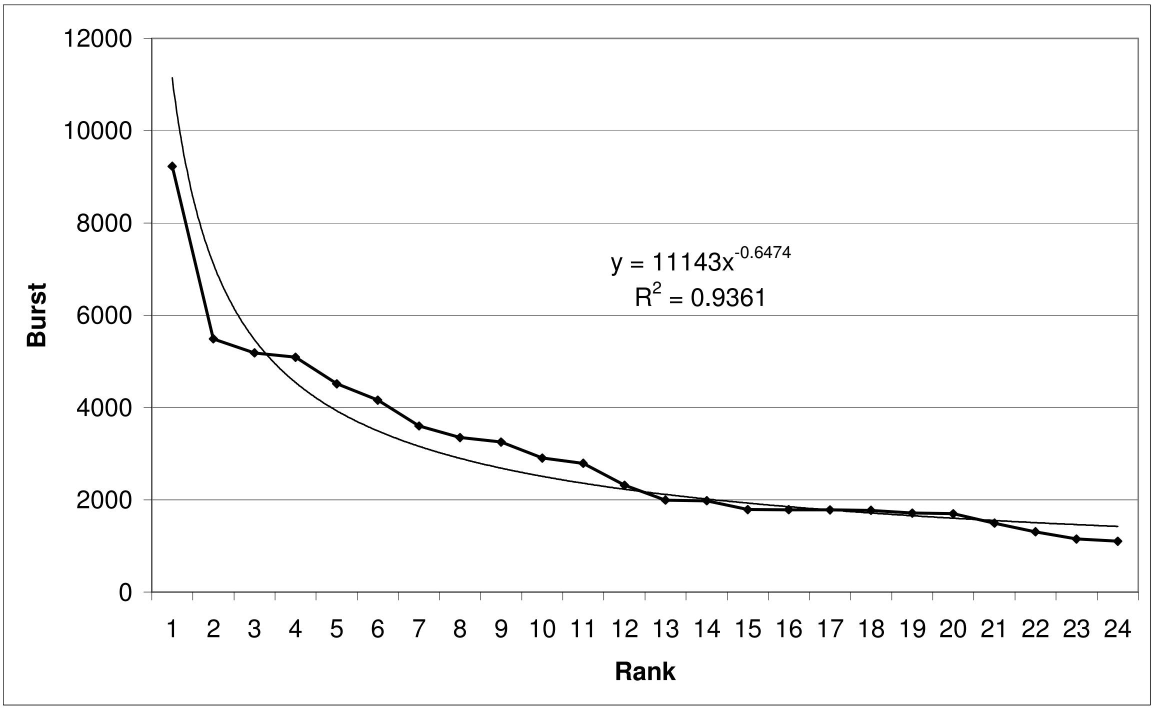


Figure 11: power law fit of urban bursts

The lack of a perfect fit is not surprising. It is well recognized that power law fit “may be only approximate in practice, and may hold only over a bounded range” (Gabaix, 2008). It has become accepted to remove the very many small centers to improve the fit.

Discussion

The study of the model suggests that the adversarial interactions between planning authorities and developers lead to the discontinuous in space and time release of lands for development and, thus, leads to the creation of polycentric urban structure.

The rules that govern the planner’s behavior lead to the emergence of small clusters of buildings of various heights in peripheral sites. When such clusters become concentrated enough, city planners realize that the slow high-rise building permissions dripping created the seed of a new urban sub-center, and that the specific peripheral zone will become urbanized. The municipality then relaxes the characteristic time constraints in that zone, setting it lowest in the new cluster, and gradually increasing it outwards. A large set of new locations where $\tau < T$ is created, and therefore new opportunities both for patient and impatient developers arise. Both devote themselves to the new sub-center development, giving rise to a construction boost which pushes the excess of demand far below the threshold. The increased activity continues until the new sub-center is fully developed, followed by a new stagnation period, when the whole process is expected to happen again.

The burst phenomenon is due to the fact that planning authorities are bounded by land-use zoning and otherwise resist granting building permits for massive developments e.g, at the city’s edge. Population growth and consequent excess demand for housing that does not receive response within the city’s boundary creates pressure for granting of building permits beyond the planned zones and leads to the establishment of new centers of urban development. Once the building process starts, the initial big developer is joined by others, including small developers, and a significant sized burst occurs.

The timing of the peaks and troughs in construction activities depend on the relative number of the two types of developers. The land purchases by the big developers and population growth are similar to accumulation of energy in physical systems. Planning authorities, in response to market pressures, release this energy. Just as in the sand-pile, the energy of population is accumulated continuously, but can be "spent" in two ways: (1) as a result of construction by impatient developers that follows standard long-term zoning and CAT; (2) purchase and construction activity initiated by the patient developers and followed by the impatient developers. The first process resolves the routine problem of dwelling demand and is hardly perceptible to the system observer. The urban changes of a second type look like bursts and lead to qualitative changes in the urban pattern. Changes in the behavior of planning authorities in response to accumulated urbanization pressure leads to qualitative changes in the spatial structure of cities. Thus, it is the behavior of planning authorities that plays a key role in the formation of polycentric urban structures.

The periodic formation of new urban centers is influenced by the sensitivity of planning authorities to growing excess demand for housing (alpha in the model), and the pressure that developers have to impose on the municipality (beta in the model). The lower are the alpha and beta the stronger is the resistance of the planner to demand and to developers' pressure and, consequently, the longer the time until the new seed of urban activity will emerge. The demand will be transferred to other cities and the investment of the big developers nearby will not be returned.

There is a need for empirical analysis of the behavior of planning authorities and developers in order to reveal the nature of these parameters. In particular, there is a need to examine whether alpha and beta are constant or the pressure of demand and developers cause their growth and thus, the emergence of new urban centers becomes inevitable.

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