doi:10.1068/b37073

# Cities in competition, characteristic time, and leapfrogging developers

# Dani Broitman, Daniel Czamanski

Faculty of Architecture, Technion—Israel Institute of Technology, Haifa 32000, Israel; e-mail: danib@techunix.technion.ac.il, danny@czamanski.com Received 29 May 2010; in revised form 13 November 2011

Abstract. In a recent paper Czamanski and Roth (2011 Annuls of Regional Science 46 101-118) demonstrated that, because the profitability of construction projects is influenced by variations in the time incidence of costs and revenues, despite declining willingness to pay and land gradients with distance from central business districts, profitability can experience local maxima away from urban centers. The time until the realization of revenues was termed 'characteristic time'. Its size is the result of planning polices and can lead to leapfrogging and scattered development, especially when interest rates are low or negligible. We explained this result by modeling the simple behavior of developers in the context of a single linear city. In this paper we consider the case of two municipalities with different development policies and characteristic time functions. We explore local maxima in profitability, typical of disequilibrium situations, especially during periods when cities are growing. Myopic assumptions, in the sense that each city is interested only in what happens on its side of the municipal boundary, can easily lead to unintended leapfrogging. Competition between cities can result in intentional leapfrogging or in spatially concentrated development, depending on the policy objectives. We extend the analysis further and consider qualitatively different cities that give rise to different gravitytype forces and differences in willingness to pay. The demand and supply sides of the building market are integrated into the model. The additional considerations can lead to various patterns of scattered development capable of explaining the spatial structure of metropolitan areas.

Keywords: urban spatial dynamics, sprawl, characteristic time, high-rise buildings

# **1** Introduction

Urban spatial development is the subject of many books and hundreds of papers. In simple Alonso-type models (Alonso, 1964; Fujita, 1989; Mills, 1967) people and activities that agglomerate in cities to benefit from mutual proximity compete for space and locations (Duranton and Puga, 2004; Zenou, 2009). Preferences are represented by a demand for geographic locations in relation to city centers. The winners are willing and able to pay more than others for this proximity. As a result land rents at the city centers should be high and should decline from this location outwards. We should observe monotonically decreasing land rents and density as the distance from the urban centers increases.

Empirical tests of these models utilized crude tools and averaged data to estimate rent gradients, and decreasing exponential functions to depict urban spatial structure (Alperovich and Deutsch, 2000). The resulting empirical regularity is true only at the crudest resolution. Even casual empiricism suggests that at a finer resolution the evidence is quite different. There is a growing body of evidence that urban spatial dynamics are discontinuous in space and nonuniform in time (Benguigui and Czamanski, 2004; Benguigui et al, 2000; 2001a; 2001b; 2004; 2006). The evidence suggests that in each period there is a proportional addition of buildings in each height category. The findings indicate a seemingly random spatial dynamics of high-rise buildings (Benguigui et al, 2008).

It is noteworthy that the classical models focused only on the demand side of the market. The vast majority of the various elaborations and applications ignored the supply side, the considerations of planners and developers, and the characteristics of locations. The classical models assumed homogeneity of consumers and of producers except for one parameter—the consumers' willingness to pay (WTP) for proximity to the urban center, or secondary centers in the urban space.

If the elasticity of the demand functions was low relative to the elasticity of the supply, then the responsiveness of the quantities of buildings to changes in demand parameters would be high. This is not the case in typical real-estate markets and definitely not the case in the high-rise buildings market. In these markets there are many consumers and very few suppliers or developers. The influence of the consumers on quantity and prices is low and therefore the aggregate demand curve is very flat and elastic. The supply curve is much less elastic. These market conditions yield high dependence on the supply side and its characteristics. Uncertainties of supply, which represents the behavior of developers and planners, can cause high and unexpected fluctuations in the quantity of high-rise buildings. This in turn can influence and support random statistical and spatial processes. There is a need for an alternative approach that can explain the basic factors responsible to these uncertainties.

Following the seminal paper by Krugman (1991) and the birth of new economic geography, uneven spatial development became an object of renewed and intensive interest. Spatial evolution has been studied at a variety of scales. At each scale it is related to different agglomeration forces that create spatial inequality. According to Fujita and Thisse (2008, page 109) the underpinning forces are often the result of "strong tensions between different political bodies or jurisdictions." The jurisdictions create the rules of the playing field within which preferences of individuals lead to decisions and create spatial order. In the urban context, the land market constitutes the playing field. It serves to allocate both economic agents and activities across space.

In this research we follow Henderson and Venables (2008) and argue that developers consider the preferences of consumers and that the behavior of builders determines the spatial structure of cities. This behavior can reflect various objective functions and spatial conditions within which the decisions are made. Following Czamanski and Roth (2011) we present a simple model of the behavior of developers. We then study the repercussions of the developers' behavior for the spatial structure of cities under various assumptions concerning the environments within which their decisions are made.

In a companion paper we discussed the case of two adjacent jurisdictions, each with its own characteristic time function, myopic behavior of planners, and lack of rivalry among the jurisdictions (Broitman and Czamanski, 2011). We now extend this theoretical framework. We study both the supply and demand sides of the housing market within a linear space bounded by two qualitatively different cities. The supply side is represented by the developer's behavior, influenced by planning regulations, and the demand side is expressed by the willingness to pay for a house as a function of the gravity-type forces.

In section 2 we present the motivation for the analyses that follow by means of data for Tel Aviv indicating an urban structure that is discontinuous in space and nonuniform in time. In section 3 we present the basic model of developers' behavior, explore the consequences of different developers' strategies, and describe the case of two neighboring, similar cities. In section 4 we describe the case of two qualitatively different neighboring cities without competition. In section 5 we present the case of competition among unequal cities. Some conclusions follow in the final section.

#### 2 The 3D structure of Tel Aviv

Tel Aviv is the second biggest city in Israel and is part of a large metropolitan area (Gush Dan) that consists of a number of big municipalities. The initial development of Tel Aviv took place in the early years of the 20th century. Until the 1970s Tel Aviv was a flat city with very few high-rise buildings. In the 1970s, 1980s, and mainly at the end of the 1990s a large number of high-rise buildings appeared.

Using GIS data of building layers for the years 1972, 1986, and 2003 and a definition of high-rise buildings as buildings with height  $\geq 25\,$  m we found evidence for the assertion that the Tel Aviv urban spatial dynamics are discontinuous in space and nonuniform in time. We found that in each year there is a proportional addition of buildings in each height category, indicating a seemingly random dynamics of high-rise buildings.

In each of the three years the distribution of all building heights displays twin peaks, and a moderate tendency for the relative number of low buildings to decrease as a function of time (Roth, 2009). This can be seen in the histograms for the three years presented in figure 1. While the general shape of the height histograms remained unchanged during the three years, there is significant horizontal movement representing the transition to taller buildings in Tel Aviv (see figure 2). The figure consists of three distribution curves for all heights scaled according to the frequency axis. The curves display two local maxima and one local minimum:



Figure 1. Distribution of all building heights in Tel Aviv. (a) 1972; (b) 1986; (a) 2003.



Figure 2. Scaled height-distribution curves for all heights of buildings in Tel Aviv in 1972, 1986, 2003.

We studied the spatial dynamics of building heights in Tel Aviv using a grid of cells (Golan, 2009). For each cell we calculated the average height of buildings. Figure 3 illustrates this classification according to five natural intervals. Based on a number of clustering tests we found only weak and weakening evidence for clustering of high-rise buildings. Thus, the 'average nearest neighbor' (ANN) clustering measure (Clark and Evans, 1954)



Figure 3. The dynamics of heights in Tel Aviv: (a) 1972; (b) 1986; (c) 2003.

displays monotonic increase and weakening of the clustering of high-rise property over time (ANN = 0.31 in 1972, 0.36 in 1986, and 0.39 in 2003). Similar results were obtained using the Moran index (MI) (Moran, 1950), showing a positive tendency to cluster, weakly autocorrelated due to the spatial heterogeneity of the heights (MI = 0.03 in 1972, 0.07 in 1986, and 0.04 in 2003).

In conclusion, the evolution of the city is comprised of two interrelated processes: the time evolution of heights and their spatial spread. Both processes do not accord well with Alonso-type models.

#### 3 Developers' behavior, characteristic time, and similar cities

At the heart of our approach is a simple conception of land developers' behavior and of the environment within which they function. We assume a linear city. Figure 4 presents the stylized facts. The central business district (CBD) of one city A is at  $x_A$ , and that of another city B is at  $x_B$ . The boundary between the two cities is at  $x_L$ . We begin with the case of one city, city A. The developers' problem is to find an optimal location  $x^*$  and optimal height  $h^*$  that leads to profit maximization. The developer's objective function (Czamanski and Roth, 2011) is:

maximize FV
$$(t = \tau) = -I(x, h)(1 + n)^{\tau} - C(h) + P(x)h,$$
 (1)  
x, h

such that

 $\tau = \tau(x,h) \; ,$ 

where  $\tau$  is the characteristic time and accounts for the time from the moment of acquisition of property rights by a developer until the realization of returns. I(x, h) represents the land price as a function of location x and building rights expressed as height of buildings, h. The discount rate is r. The overnight building cost is a function of building height and is expressed as C. Finally, P(x) is the WTP of buyers at location x.



Figure 4. The spatial structure of linear cities.

Analyzing the first-order conditions for the optimal location and height in the single-city model, Czamanski and Roth (2011) arrived at the following conclusions:

(1) Leapfrogging of heights occurs when

$$\frac{\partial h^*}{\partial \tau} > 0$$
, and  $\frac{\partial \tau}{\partial x^*} > 0$ , and therefore  $\frac{\partial h^*}{\partial x^*} = \frac{\partial h^*}{\partial \tau} \frac{\partial \tau}{\partial x^*} > 0$ . (2)

(2) There are three distinct possibilities for the sign of  $\partial \tau / \partial x^*$ :

$$\frac{\partial \tau}{\partial x^*} = 0, \qquad \frac{\partial \tau}{\partial x^*} > 0, \quad \text{or } \frac{\partial \tau}{\partial x^*} < 0 \; .$$

(3) If  $(\partial \tau / \partial x^*) = 0$  then in equation (2)

$$\frac{\partial h^*}{\partial x^*} = \frac{\partial h^*}{\partial \tau} 0 = 0 \; .$$

In this case the optimal height does not depend on the distance from the CBD. (4) If,  $\partial \tau / \partial x^* > 0$  then the sign of  $\partial h^* / \partial \tau$  in equation (2) is not clear and the condition for a positive derivative that can lead to leapfrogging is a very low interest rate, such as may occur during periods of recession.

(5) If  $(\partial \tau / \partial x^*) < 0$  it can be shown that  $(\partial h^* / \partial \tau) > 0$  and therefore

$$\frac{\partial h^*}{\partial x^*} = \frac{\partial h^*}{\partial \tau} \frac{\partial \tau}{\partial x^*} < 0$$

In this case,  $h^*$  is a decreasing function of the distance from the CBD. As in the previous case, leapfrogging is possible only when the interest rate is negligible.

The optimal location model analyzes a single developer's choice and assumes implicitly that this represents an average behavior. In order to test the implications of different developers' behavior, a simple agent-based model was created, simulating land purchasing and development activities in the context of a single city. If all the developers were constrained to choose only profitable locations with low characteristic time, an Alonso-type city emerges, as shown in figure 5(a). Grey cells represent undeveloped sites. Dark cells represent intensively developed areas.



Figure 5. Two-dimensional city model with homogenous developers (a) and speculative behavior allowed (b).

With the help of the same model we study the repercussions when developers are allowed to speculate, purchasing land parcels with very high characteristic time. They benefit from low land costs that reflect the fact that development will not be allowed easily and it will take place after long waiting times. On the other hand, as more of such locations are being purchased, pressure rises on the planning authorities to allow development in those marginal sites. If flexible planning policies are introduced into the model, the characteristic time can be lowered in zones where several parcels are waiting for development. If this happens, the speculative developers take advantage of a disequilibrium situation and build high-rise buildings exploiting the gap between high willingness to pay and cheap land. Later, other developers are attracted to the zone by the lowered characteristic time, but then the gap between costs and revenues shrinks. This type of situation leads to the creation of subcenters far from the initial CBD. The speculative developers act as the workhorses leading to the emergence of a new subcenter. Should they succeed in changing the planning policy in that zone others follow them. A typical result is shown in figure 5(b). In this case several subcenters, each with Alonso-type structure, emerge.

However, the rudimentary model that is the basis of this paper is enough to illustrate that even if only an average developer is assumed, interaction between two cities and their respective planning policies are conducive to several cases of leapfrogging, scattered development, or subcenter creation.

In the case of two cities, the CBD of city B is located at  $x_B$ .  $x_L$  is the administrative boundary between the two municipal areas, A and B (see figure 4). For the present purpose the location of this boundary is insignificant. It is assumed that it was defined as part of an historical, political process. The segment ( $x_L$ - $x_A$ ) and the segment ( $x_B$ - $x_L$ ) are not necessarily equal.

In each municipality the characteristic time is determined by an independent planning authority. We presume that in a Tiebout-style world (Tiebout, 1956), each planning authority reflects the preferences of its self-selected residents. The characteristic time in each municipality pertains only to its own territory. As in the single municipality case,  $\tau$  is a function of the distance from the respective CBD. It is also a function of the intensity of the proposed development. Thus, it is an increasing function of building height. In other words:

$$\tau = \tau(x, h), \begin{cases} \tau^{A}(x, h), & \text{if } x \in [x_{A}, x_{L}], \\ \tau^{B}(x, h), & \text{if } x \in (x_{L}, x_{B}]. \end{cases}$$
(3)

It is assumed that the developers have precise information about the characteristic time at each location over the segment [ $x_A$ ,  $x_B$ ]. Various situations may occur, reflecting all the possible combinations of  $\tau_x^A$  and  $\tau_x^B$  (positive, negative, or zero functions). Assuming myopic behavior (in the sense that each municipality defines the characteristic time in its respective area of influence independently) or full awareness lead to much more diverse scenarios.

Those scenarios were analyzed elsewhere (see Broitman and Czamanski, 2011) for the case of similar cities and restricted to the developer's behavior under different characteristic times. There the demand side was not included in the model. The conclusions were that myopic assumptions can easily lead to unintended leapfrogging, whereas competition between the cities, including the case that each city takes into consideration processes in the entire region, can result in intentional leapfrogging or in spatially concentrated development, depending on the policy objectives of the authorities. Additional scenarios of collaboration between authorities with different goals are also feasible.

#### 4 Dissimilar cities—no competition

We assume that cities A and B are qualitatively different, in terms of population, size, and purchase power. A is assumed to be a developed city, with high population (at least one order of magnitude greater than B's population), and consequently offering diverse urban economic, social, and cultural amenities (employment, manufacturing, and business as well as consumer facilities and educational options). B is assumed to be a small city, much less populated than A and offering a narrow spectrum of urban amenities, but maybe offering better country-side amenities than A (open spaces, less agglomeration). Furthermore, it is assumed that A and B are located at a commuting distance from each other, meaning that it is possible to live in B and work and consume cultural products at A.

From a consumer's point of view, the WTP for a house in this linear model is mainly a function of the distance from the main CBD as assumed by monocentric and multicentric urban models (Alonso, 1964; Mills, 1967; Muth, 1969; Wheaton, 1982). Since the linear space is bounded by two business districts, each exerts an attraction that operates along the entire  $[x_A, x_B]$  segment. The strongest attraction forces operate on the edges themselves, being much more powerful in relation to the CBD of A, since A is the bigger city. Along the  $[x_A, x_B]$  segment they are expected to decrease as the distance from A increases (Osland, 2008; Osland and Thorsen, 2005). B's attraction force eventually rises as the proximity to the CBD of B increases. The WTP for housing is assumed to be the emerging result of those gravity forces, and its graph is sketched in figure 6.



Figure 6. Willingness to pay (WTP) for housing in the linear model. LWTP = least WTP.

In order to facilitate the discussion in what follows, the developers' objective function defined in equation (1) is redefined as a simpler revenue-minus-cost function. The first component in the function is  $I(x, h)(1 + r)^{\tau}$ . The initial investment (I) is assumed to be a decreasing function of x, and an increasing function of h. The characteristic time increases the investment component since it imposes a waiting interest on it. The characteristic time can be an increasing function of x, and is an increasing function of h. The investment cost is augmented if  $\tau_x > 0$  and diminishing if  $\tau_x < 0$ .

The second component C(h) is the overnight cost, an increasing function of the building height  $(C_h > 0)$ . Moreover, its slope tends to increase (its second derivative  $C_{hh} > 0$ ). This is because building a marginal floor should be more expensive than building the previous one (otherwise the result is always an optimal infinite height). The last component is the developers' revenue, namely P(x)h, or the WTP for a house at location x, times the number of floors. A developer will build an additional floor until C(h + 1) - C(h) = P(x).

Although the initial investment is an increasing function of h, it is assumed to be negligible compared with the marginal floor building cost. Therefore, the height parameter

itself is simplified to be proportional to the WTP.<sup>(1)</sup> This simplification allows us to define a developer cost function  $C_D(x)$  depending on x only. The characteristic time is embedded in it, raising it when  $\tau_x > 0$  or diminishing if  $\tau_x < 0$ .

If a particular fixed height (h = K) is assumed for the analysis, the building costs are fixed and can be ignored since they are the same for each location. Using the following substitutions

$$-C_{\rm D}(x) = -I(x)(1+r)^{\tau}, \qquad (4)$$

such that

$$C_{\rm D} = C_{\rm D}[\tau(x)] ,$$
  

$$P(x)K = W(x) ,$$
(5)

(W = willingness to pay). The developer's profit objective function defined in equation (1) was simplified as

$$maximize \pi = -C_{\rm D}(x) + W(x), \qquad (6)$$

 $x \in [x_A, x_B]$ 

such that

 $C_{\rm D} = C_{\rm D}[\tau(x)] \; .$ 

The new objective function  $\pi$  defined in equation (6) can be visualized as the difference between the developer's cost function and the WTP along the [ $x_A$ ,  $x_B$ ] segment as in figure 7.



Figure 7. Willingness to pay (WTP) and developer costs ( $C_D$ ) in the linear model. LWTP = least WTP.

The assumption that  $\tau_x^A = 0$  and  $\tau_x^B = 0$  means that planning policies are place independent in the case of both cities. In other words, in each city characteristic time does not depend on location, nor is it dependent on the optimal height of proposed building projects. The developer's decision concerning where and at what intensity to build is not influenced by characteristic time imposed by the municipalities (since it is the same at each location). Clearly, in this case, the maximization problem is solved at CBD A, because the WTP is greatest.

In this case the regional development will follow the pattern depicted in figure 8. First, all the available sites will be developed from CBD A outwards to  $x_1$  (arrow 1). In a second phase, the segments  $[x_1, x_{LWTP}]$  and  $[x_{LWTP}, x_B]$  will be developed intermittently according to the slope of the WTP function (following arrows 2 and 3), finishing in  $x_{LWTP}$  only when running out of space.

<sup>(1)</sup>Another way to look at our simplification is to assume that we are considering the profitability of a single building of a particular height at different locations.



**Figure 8.** Willingness to pay (WTP) and constant characteristic scenario.  $C_D$  = developing costs. LWTP = least WTP.

A different scenario arises when one municipality implements a constant characteristic time policy, and the other implements either an increasing or decreasing characteristic time as a function of the distance from the CBD. If the developed city A wishes to avoid leapfrogging (and therefore in its territory  $\tau_x^A > 0$ ), while in city B area  $\tau_x^B = 0$ ; the result may be an undeveloped gap (as segment  $[x_1, x_L]$  in figure 9). This is because A's policy increases the developer's costs, but only until A's administrative boundary  $(x_L)$ . In this case, A's myopic behavior can lead to an outcome opposite to that originally intended—scattered development arising beyond its municipal boundary.



**Figure 9.** No competition,  $\tau_x^A > 0$  and  $\tau_x^B = 0$ . (See text and earlier figures for definitions.)

If A implements a policy aimed at spreading its development, for reasons such as excessive agglomeration in its CBD, then  $\tau_x^A < 0$ , and the results will be according to A's policy. However, this policy can lead to an unintended overdevelopment of B. If developers' profit in A's CBD diminishes enough, the highest profits can be achieved in B's CBD. Again, myopic behavior can lead to unexpected results, unless the policy is coordinated between both municipalities and the outcome is welcomed by both. Figure 10 describes this scenario.

Inversely, an increasing characteristic time implemented by the smaller city B, may lead to a concentrated development in A's territory. If  $\tau_x^B > 0$  and no specific policy is implemented by A ( $\tau_x^A = 0$ ), development can be severely restrained on [ $x_L, x_B$ ], as can be seen in figure 11. On the other hand, if the policy implemented by city B is the opposite ( $\tau_x^B < 0$ ) the result is an incentive to leapfrogging development in segment [ $x_L, x_B$ ], as depicted in figure 12.

Since the willingness to pay near the big city is greater than in any other place in the linear model, increasing development costs in B's territory near the administrative border will restrain development to segment  $[x_A, x_L]$  (figure 11). On the other hand, increasing development costs in B's center may be an incentive to develop the border zone in B's area.



**Figure 10.** No competition,  $\tau_x^A > 0$  and  $\tau_x^B = 0$ . (See text and earlier figures for definitions.)



**Figure 11.** No competition,  $\tau_x^{\rm B} > 0$  and  $\tau_x^{\rm A} = 0$ . (See text and earlier figures for definitions.)



**Figure 12.** No competition,  $\tau_x^B > 0$  and  $\tau_x^A = 0$ . (See text and earlier figures for definitions.)

Whether or not it is a desirable outcome depends on the big city's development policies and goals. Again, a myopic behavior (in this case by city B) can lead to unexpected outcomes on the other side.

## 5 Competition among dissimilar cities

In this section we assume that the qualitatively different two cities are in rivalry. If the development policies of the municipalities are similar, they will compete in order to pursue their goals. If an urban concentration policy is defined by both municipalities, then both  $\tau_x^A > 0$  in  $[x_A, x_L]$  and  $\tau_x^B > 0$  in  $[x_L, x_B]$ , in order to discourage development far from the respective CBDs, raising costs as the distance increases. In this case the respective cost and WTP curves will look as in figure 13. Development is profitable mainly in areas near the big city CBD and, as a second option, in areas near the small city CBD, where development in the hinterland in strongly discouraged.



**Figure 13.** Rival cities,  $\tau_x^A > 0$  and  $\tau_x^B > 0$ . (See text and earlier figures for definitions.)

If we assume that other policies are adopted by both municipalities, for example, spreading the development far from the respective CBDs, then both  $\tau_x^A < 0$  in  $[x_A, x_L]$  and  $\tau_x^B < 0$  in  $[x_L, x_B]$ . The effects of decreasing characteristic time with the distance from the center on the cost function are twofold: raising the cost near the CBDs and lowering it in the hinterland, as seen in figure 14.



**Figure 14.** Rival cities,  $\tau_x^A \le 0$  and  $\tau_x^B \le 0$ . (See text and earlier figures for definitions.)

Though the development-spreading goal can be achieved from the point of view of both municipalities, potential conflicts can still arise due to the mismatch between the administrative boundary and the point where the WTP is lowest (in other words, due to the mismatch between the economic attraction of city A and the territorial arrangements). Development in the segment [ $x_L$ ,  $x_{LWTP}$ ], despite being welcomed by both municipalities, contributes (via property taxes, for example) much more for city B than for city A. In that segment a new building is constructed because of A's attraction force, but B benefits from the outcome (assuming that both see leapfrogging as a positive result).

If the cities' development policies differ, while assuming that both municipalities are aware of the tendencies and processes occurring in the entire  $[x_A, x_B]$  segment, different conflicts can occur, depending on the location of the administrative boundary, the relative economic strength of the cities, and the extent of cooperation or competition between them.

If the big city A is willing to spread its development outwards, it can implement a characteristic time that encourages developers to build far from the center ( $\tau_x^A < 0$ ), but only in segment [ $x_A$ ,  $x_L$ ]. At the same time, city B is willing to conserve its open space and therefore implements a restrictive policy ( $\tau_x^B > 0$ ) in [ $x_L$ ,  $x_B$ ], affecting the cost function, as can be seen in figure 15.

Under these conditions the A border side (left of  $x_L$ ) is expected to develop faster and create a leapfrogging pattern in A's territory. Across the border and as far as city B's edge,



**Figure 15.** Rival cities,  $\tau_x^A < 0$  and  $\tau_x^B > 0$ . (See text and earlier figures for definitions.)

there are expected to be no buildings. City A is able to use all its territory in order to pursue its goals, but on the other hand (if  $x_L$  is too close to A) this area might be not enough for this. City B may prefer to see most of  $[x_A, x_B]$  as open space, but is able to exert influence only on segment  $[x_L, x_B]$ . An alternative interpretation may be that both cities coordinate their policies in order to take advantage of the entire area, each one maximizing its utility according to its preferences. The problem with this interpretation is that city A is not only achieving its goals, it is free-riding on B's willingness to maintain open spaces. Dwellers in the newly developed neighborhoods near A's boundary will enjoy plenty of open space for free, whereas city B is loosing (readily) potential income. This opportunity cost to city B can lead to claims from city A.

Reversing the scenario, if the big city A is willing to concentrate its development near its CBD it can implement a characteristic time that will encourage developers to build as near as possible to the center ( $\tau_x^A > 0$ ), but only in its own administrative area, segment [ $x_A$ ,  $x_L$ ]. City B is allowing development far from its CBD, and implements a permissive policy ( $\tau_x^B < 0$ ) in [ $x_L$ ,  $x_B$ ]. The resulting cost function is depicted in figure 16.



**Figure 16.** Rival cities,  $\tau_x^A > 0$  and  $\tau_x^B < 0$ . (See text and earlier figures for definitions.)

In this scenario, the fast developing areas will be A's CBD and near the border in B's area. The result is an unintended leapfrogging pattern from the point of view of city A. The small city is taking advantage of the fact that the big city attraction is felt far away from its administrative border and is getting revenues from urban development that is located in its area due to the proximity to the big city A. City A is constraining its own spatial development in order to keep enough open spaces at its edge, but it is forced to witness fast and unintended development on the border's other side. City B is not likely to give up its revenues from property taxes and population growth for nothing. One possible solution for the conflict is compensation from city A to city B, in exchange for avoiding some development in B's area.

Such compensation should be equal to the opportunity cost lost by B when cancelling a welcome urban development.

# 6 Conclusions

A simple developer's behavior model, in the context of a single city within a linear space, is able to explain leapfrogging and scattered development as a consequence of spatial variation in characteristic time, especially in times when interest rates are low or negligible (Czamanski and Roth, 2011).

If two cities are sited at opposite edges of a linear space, the same situation can arise, but additional factors play an important role and need to be considered. In this case, each city defines its own development policies that are reflected in different characteristic time functions in each territory.

Assuming that both cities are qualitatively different, a developed city with a large population and diverse economic, social, and cultural amenities and a small city at the opposite edge, additional forces and interactions between them emerge. In this scenario, mutual attraction forces will operate along the entire region, interacting with the political boundary between them, which does not necessarily reflect the gravity influences shaped by the market forces.

If each city is interested only in what happens on its side of the boundary, a behavior we term 'myopic', leapfrogging patterns can easily develop. Even full awareness, in the sense that each city takes into consideration processes in the entire region, can result in intentional leapfrogging created by competition between the cities. However, in this case, spatially concentrated development is possible, depending on the policy objectives of the authorities. Additional scenarios of collaboration between authorities with different goals are also feasible.

The model simulates processes occurring during a phase when both cities are growing, and the land and residential markets are not in equilibrium. In the long run an equilibrium stage, where profitability is the same everywhere, will eventually be reached, but our interest is focused on the long period (years or decades) in between. In such disequilibrium situations, local maxima in profitability are possible, for example in some of the scenarios depicted above. Those local maxima can be intentionally or unintentionally created by city authorities, emerging from non-monotonic time incidences on development along their territories. Since profitability is highly influenced by time incidence, developer's location decisions can be very different from the results expected by models assuming monotonic profitability decline from the CBD.

Acknowledgement. The authors wish to thank the Israel Science Foundation for its generous support of this research.

## References

Alonso W, 1964 Location and Land Use (Harvard University Press, Cambridge, MA)

- Alperovich G, Deutsch J, 2000, "Urban non-residential density functions: testing for the appropriateness of the exponential function using a generalized Box–Cox transformation function" *Annals of Regional Science* **34** 553–568
- Benguigui L, Czamanski D, 2004, "Simulation analysis of the fractality of cities" *Geographical Analysis* **36** 69–84
- Benguigui L, Czamanski D, Marinov M, Portugali Y, 2000, "When and Where Is a City Fractal?" Environment and Planning B: Planning and Design 27 507–519
- Benguigui L, Czamanski D, Marinov M, 2001a, "The dynamics of urban morphology: the case of Petah Tikvah" *Environment and Planning B: Planning and Design* **28** 447–460
- Benguigui L, Czamanski D, Marinov M, 2001b, "City growth as a leap-frogging process: an application to the Tel Aviv Metropolis" *Urban Studies* **38** 1819–1839

- Benguigui, L, Czamanski D, Marinov M, 2004, "Scaling and urban growth" Journal of Modern Physics C 15 989–996
- Benguigui L, Blumenfeld E, Czamanski D, 2006, "The dynamics of urban morphology" Environment and Planning B: Planning and Design **33** 269–284
- Benguigui L, Czamanski D, Roth R, 2008, "Modeling cities in 3D: a cellular automaton approach" *Environment and Planning B: Planning and Design* **35** 413–430
- Broitman D, Czamanski D, 2011, "Developers' choices under varying characteristic time and competition among municipalities" *Annals of Regional Science* **46** 101–118
- Clark P, Evans F, 1954, "Distance to nearest neighbor as a measure of spatial relationship in populations" *Ecology* **34** 445–453
- Czamanski D, Roth R, 2011, "Characteristic time, developers' behavior and leapfrogging dynamics of high-rise buildings" *Annals of Regional Science* **46** 101–118
- Duranton G, Puga D, 2004, "Micro-foundations of urban agglomeration economies", in *Handbook* of *Regional and Urban Economics*, edition 1, volume 4, Ed. J V Henderson, J F Thisse (Elsevier, Amsterdam) pp 2063–2117
- Fujita M, 1989 Urban Economic Theory, Land Use and City Size (Cambridge University Press, Cambridge)
- Fujita M, Thisse J F, 2008, "New economic geography: an appraisal on the occasion of Paul Krugman's 2008 Nobel Prize in Economics" *Regional Science and Urban Economics* **39** 109–119
- Golan E, 2009, "3D urban morphology", Master's thesis, Technion—Israel Institute of Technology, Haifa
- Henderson J V, Venables A J, 2008, "The dynamics of city formation" *Review of Economic Dynamics* 12 233–254
- Krugman P R, 1991, "Increasing returns and economic geography" *Journal of Political Economy* **99** 483–499
- Mills E, 1967, "An aggregative model of resource allocation in a metropolitan area" *American Economic Review* **57** 197–210
- Moran P, 1950, "Notes on continuous stochastic phenomena" Biometrika 37 17-23
- Muth R, 1969 Cities and Housing (University of Chicago Press, Chicago, IL)
- Osland L, 2008 Spatial Variation in Housing Prices: Econometric Analyses of Regional Housing Markets PhD thesis, ....., University of Bergen
- Osland L, Thorsen I, 2005, "Effects on housing prices of urban attraction and labor market accessibility" *Working Papers in Economics* **17** 105, University of Bergen, http://www.uib.no/Filearchive/filetopic\_No.%2017-05.pdf
- Roth R, 2009 3D Urban Dynamics PhD thesis, Technion-Israel Institute of Technology, Haifa
- Tiebout C M, 1956, "A pure theory of local expenditures" *The Journal of Political Economy* **64** 416–425
- Wheaton W,1982, "Urban residential growth under perfect foresight" *Journal of Urban Economics* **12** 1–21
- Zenou Y, 2009 Urban Labor Economics (Cambridge University Press, Cambridge)

© 2012 Pion and its Licensors